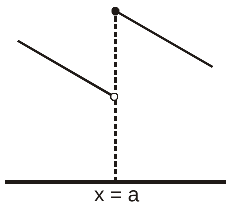
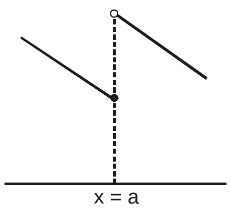
EXERCISE – III SUBJECTIVE QUESTIONS

1. Consider the following graph. In each case identify if x = a is a point of maxima, minima or neither maxima nor minima

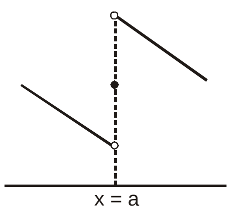
(i)



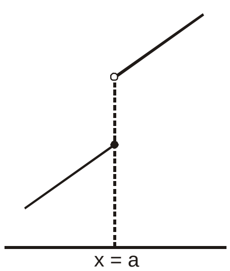
(ii)



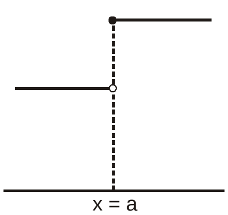
(iii)



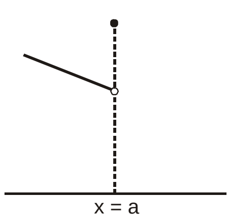
(iv)



(v)



(vi)



2. Find values of a and b such that $f(x) = \frac{a}{x} + bx$ has a minimum at point (1, 6).

3. Find the points of local maxima/minima of following functions

(i) $f(x) = 2x^3 - 21x^2 + 36x - 20$

(ii) $f(x) = -(x-1)^3 (x+1)^2$

(iii) $f(x) = x \, \ell n x$

4. Let $f(x) = \begin{cases} 2\sin x & x > 0 \\ x^2 & x \le 0 \end{cases}$. Investigate the function for maxima/minima at x = 0.

5. Find the number of critical points of the following functions.

(i) $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$; $x \in R$

(ii) $f(x) = |x-2| + |x+1|$; $x \in R$

(iii) $f(x) = \min(\tan x, \cot x)$; $x \in (0, \pi)$

6. Draw graph of $f(x) = x|x-2|$ and hence find points of local maxima/minima.

7. Find the absolute maximum/minimum value of following functions

(i) $f(x) = x^3$; $x \in [-2, 2]$

(ii) $f(x) = \sin x + \cos x$; $x \in [0, \pi]$

(iii) $f(x) = 4x - \frac{x^2}{2}$; $x \in \left[-2, \frac{9}{2}\right]$

(iv) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$; $x \in [0, 3]$

(v) $f(x) = \sin x + \frac{1}{2}\cos 2x$; $x \in \left[0, \frac{\pi}{2}\right]$

8. Let $f(x) = x^2$; $x \in [-1, 2)$. Then show that f(x) has exactly one point of local maxima but global maximum is not defined.

9. Let $f(x) = x + \sqrt{x}$. Find the greatest and least value of f(x) for $x \in (0, 4)$.

10. Let $f(x) = \begin{cases} 3-x & 0 \le x < 1 \\ x^2 + \ell nb & x \ge 1 \end{cases}$ find the set of values of b such that f(x) has a local minima at x = 1.

11. Find the points of local maxima/minima of following functions

(i) $f(x) = x + \dfrac{1}{x}$ **(ii)** f(x) = cosec x

Hence find maxima and minima values of f(x).

12. Show that $\sin^p \theta \cos^q \theta$, $p, q \in N$ attains maximum value when $\theta = \tan^{-1} \sqrt{\dfrac{p}{q}}$. Identity if it is a global maxima or not.

13. If the sum of the lengths of the hypotenuse and another side of a right angled triangle is given, show that the area of the triangle is a maximum when the angle between these sides is $\pi/3$.

14. A wire of length 20 m is to be cut into two pieces. One of these pieces is to be made into a square and other into a circle. What should be the length of the two pieces so that the combined area of square and circle is minimum.

15. Show that the volume of the greatest cylinder which can be inscribed in a cone of height 'h' and semi–vertical angle α is $\dfrac{4}{27} \pi h^3 \tan^2 \alpha$.

16. Show that the semi vertical angle of a right circular cone of maximum volume and of a given slant height is $\tan^{-1} \sqrt{2}$.

17. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible when revolved around one of its side.

18. The fuel charges for running a train are proportional to the square of the speed generated in m.p.h. & costs Rs. 48/– per hour at 16 mph. What is the most economical speed if the fixed charges i.e. salaries etc. amount to Rs. 300/- per hour.

19. The three sides of a trapezium are equal each being 6 cms long, find the area of the trapezium when it is maximum.

20. A sheet of poster has its area 18 m^2. The margin at the top & bottom are 75 cms and at the sides 50 cms. What are the dimensions of the poster if the area of the printed space is maximum ?

21. Find the values of 'a' for which the function

$f(x) = \dfrac{a}{3} x^3 + (a + 2) x^2 + (a- 1) x + 2$ possess a negative point of minimum.

22. A figure is bounded by the curves, $y = x^2 + 1$, $y = 0$, $x = 0$ & $x = 1$. At what point (a, b), a tangent should be drawn to the curve $y = x^2 + 1$ for it to cut off a trapezium of the greatest area from the figure.

23. Find the polynomial f(x) of degree 6, which satisfies

$\underset{x\to 0}{\text{Limit}} \left(1 + \dfrac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at x = 1 and local minimum at x = 0 & 2.

24. Find the volume of the largest cylinder that can be inscribed in a sphere of radius 'r' cm.

25. A telephone company has 500 subscribers on its list and collects fixed charges of Rs. 300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Rs. 1 in the charge, one subscriber will discontinue find the charge per subscriber that will maximize the income of the company.

26. A factory D is to be connected by a road to a straight railway line on which a town A is situated. The distance DB of the factory to the railway line is $5\sqrt{3}$ km, the length AB of the railway line is 20 km. Freight charges on the road are twice the freight charges on the railway. At what point P on the railway line should the road DP be connected so as to ensure minimum cost of transporting goods from factory to town.

27. Find the cosine of the angle at the vertex of an isosceles triangle having the greatest area for the given constant length ℓ of the median drawn to its lateral side.

28. A tangent to the curve $y = 1 - x^2$ is drawn so that the abscissa x_0 of the point of tangency belongs to the interval (0, 1]. The tangent at x_0 meets the x-axis and y-axis at A & B respectively. Then find the minimum area of the triangle OAB, where O is the origin.

29. A cone is made from a circular sheet of radius $\sqrt{3}$ by cutting out a sector and keeping the cut edges of the remaining piece together. Then find the maximum volume attainable for the cone.

30. A closed rectangular box with a square base is to be made to contain 1000 cubic feet. The cost of the material per square foot for the bottom is 15 paise, for the top 25 paise and for the sides 20 paise. The labour charges for making the box are Rs. 3/-. Find the dimensions of the box when the cost is minimum.

31. Discuss the global maxima and global minima of

$f(x) = \arctan x - \frac{1}{2}\ell n\ x$ on $\left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$.

32. From a fixed point P on the circumference of a circle of radius a, the perpendicular PR is drawn to the tangent at Q (a variable). Then find the maximum area of ΔPQR.